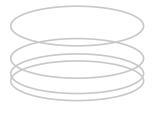


L • GARDE INC. CORPORATE PRESENTATION

Sailcraft Coodinate Systems and Format for Reporting Propulsive Performance

Billy Derbes, David "Leo" Lichodziejewski, Jordan Ellis, Daniel Scherres



SAILCRAFT COORDINATE SYSTEMS AND FORMAT FOR REPORTING PROPULSIVE PERFORMANCE

Billy Derbes¹, David "Leo" Lichodziejewski¹, Jordan Ellis², Daniel Scheeres³

In designing solar sail structures, structural engineers must calculate structural loading and resulting deflections. Byproducts of this analysis are the integrated external solar pressure forces and moments on the mainsail and vanes, as well as mass properties. This paper proposes a standard format and set of coordinate systems for reporting forces, moments, and mass properties, as well as other items relevant to performance, including packaging efficiencies and life. Sail designers will report integrated external solar pressure forces and moments on the mainsail and on the vanes (if used), as well as mass properties. Forces and moments will be non-dimensionalized into coefficients. These are tables of point values, the independent variables being attitude relative to the sun and solar distance. The user will interpolate between the point values supplied by the sail designer. Separate solar propulsion performance tables and mass properties will be reported for the mainsail alone, and for various detached single vanes (if used). Sail designers using gimbaled control mass will provide various mast length designs. This will allow the user to mix and match mainsails with control devices of various sizes. Combined forces, moments, and mass properties for the assembled sailcraft will be calculated and transformed by the user. Users can then insert attitude or vane commands and propagate the resulting solar sail trajectories and/or attitudes.

INTRODUCTION

First, reference coordinate systems are defined, followed by discussions of factors which affect performance and how they are to be represented, then coefficient definition. Finally, reporting of life and packageability are discussed.

Copyright (c) 2004 by The American Astronomical Society

¹ L'Garde, Inc., Tustin, CA

² Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA 91107

³ University of Michigan, Ann Arbor, MI

REFERENCE COORDINATE SYSTEMS

Sailcraft Body-Fixed Coordinate System "Sail"

As mass properties are reported in body-fixed systems, forces and moments will be as well. The body-fixed coordinate system "Sail" is illustrated below. The analogy is an aircraft flying over the surface of the sun. The aircraft's belly is the reflective side. The origin is to be located by the sail designer at some well-defined point in the geometric sail center.

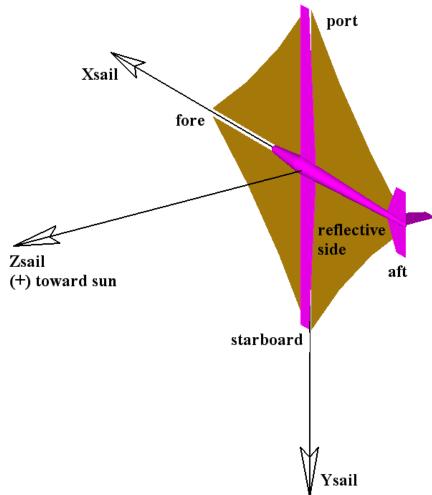


Figure 1 Body-Fixed "Sail" Coordinate System

Specification of Attitude Relative to the Sun

Two angles are to be used to identify the attitude of the Sail system relative to the sun, "Sun Incidence" and "Flatspin". "Sun Incidence" is the angle between the sail normal (Zsail) and the sail-sun line. This angle will have the greatest effect on sail forces and moments.

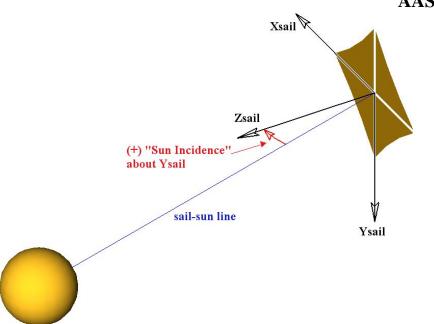


Figure 2 "Sun Incidence" Angle Between Sail Normal and the Sail-Sun Line

"Flatspin" is rotation about the sail normal, Zsail (note: NOT generally about the sail-sun line). For an idealized flat plate sail, forces and moments would not change with Flatspin. However, for a real sail, forces and moments will change somewhat due to sail billow, particularly asymmetric sail billow, as well as variations in relative beam bends.

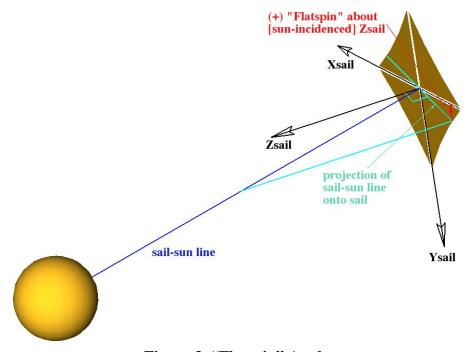


Figure 3 "Flatspin" Angle

"Top" is rotation about the sail-sun line. Beginning with Zsail coincident with the sail-sun line, the full Euler sequence is Top -> Sun Incidence -> Flatspin:

- 1. Top about Zsail, which is coincident with the sail-sun line at this point
- 2. Sun Incidence about Ysail
- 3. Flatspin about Zsail, which is no longer coincident with the sail-sun line

This is therefore a Yaw -> Pitch -> Yaw sequence.

Whatever the Sun Incidence and Flatspin angles are, the sail may be rotated arbitrarily about the sail-sun line without changing forces or moments, as expressed in the body-fixed Sail coordinate system. Top is therefore not included in the reporting of forces and moments.

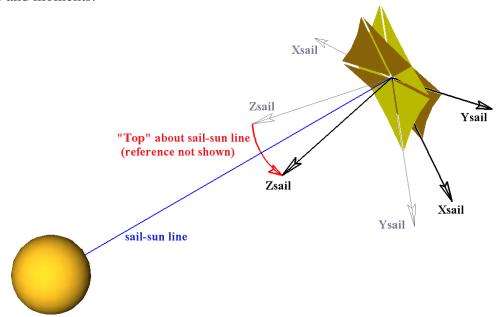


Figure 4 Body-Fixed Forces & Moments Do Not Change with "Top" Angle

Sun Incidence and Flatspin are believed to represent the propulsion of a square sail in a revealing and sensible way. Propulsion is a strong function of Sun Incidence, a weak function of Flatspin. Sail billow asymmetries are discriminated by Flatspin.

While performance is reported vs Sun Incidence and Flatspin, the user may prefer a different attitude description for interpolation due to singularity problems or other reasons. The user will convert Sun Incidence and Flatspin as necessary, but may require the sail designer to report at specific intervals of Sun Incidence and Flatspin in order not to oversample.

Note that Sun Incidence and Top are referenced to the body-fixed coordinate system Sail. They differ from McInnes's "Cone" and "Clock" (Ref. 1), which are referenced to the net solar force vector.

Example Placement in an Inertial Frame

Locating the sun-sail line in an inertial frame and specifying a reference for Top is not necessary for sailcraft performance specification, but an example is given for illustration. A suggested inertial frame is the J2000 frame. The attitude of the sail-sun line in this frame may typically be specified using "Right Ascension" and "Declination." In order to establish a reference and sense for Top and to facilitate transformations, a new coordinate system is introduced, the "Sun" system. This system moves with the sail, its origin coincident with the "Sail" system origin. Zsun always points toward the J2000 origin, along the sail-sun line. Xsun is defined as parallel to the J2000 x-y plane, and Ysun completes the orthogonal set.

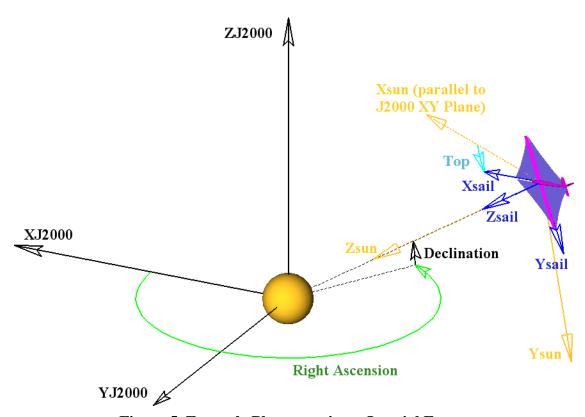


Figure 5 Example Placement in an Inertial Frame

The orientation of the Sun system relative to J2000 is defined by the following Euler sequence:

"Flip" -90° about x -> -("Right Ascension" + 90°) about y -> -"Declination" about x

The transformation matrix for a vector [V] between J2000 and Sun systems is:

To rotate from the Sun system to the Sail system:

Top about z -> Sun Incidence about y -> Flatspin about z

The transformation matrix for a vector [V] between Sun and Sail systems is:

```
[V]Sail = [R]S-Su [V]Sun

c = cos()
s = sin()

TP = Top

SI = Sun Incidence
FS = Flatspin

R11 = cTPcSlcFS -sTPsFS     R12 = sTPcSlcFS +cTPsFS     R13 = -sSlcFS
R21 = -cTPcSlsFS -sTPcFS     R22 = -sTPcSlsFS +cTPcFS     R23 = sSlsFS
R31 = cTPsSl     R32 = sTPsSl     R33 = cSl
```

One may also determine the attitude of the vehicle relative to the instantaneous velocity vector. Figure 7 illustrates the Orbit Position Reference Frame, where the x-axis is parallel to the sun-to-sailcraft line, the z-axis is parallel to the instantaneous orbital angular velocity, and the y-axis completes the orthogonal set. The attitude of both this system and the sail-fixed system relative to the inertial system are known, so the attitude of the sail relative to the Orbit Position Reference Frame is a simple subtraction.

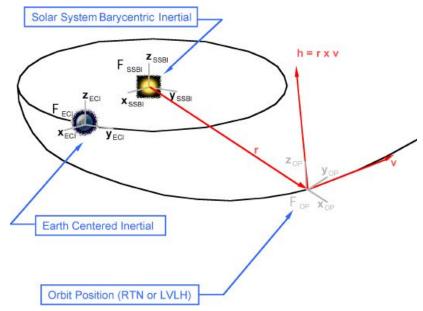


Figure 6 Orbit Position Reference Frame

Specification of Beam Tips Relative to the "Sail" System

If vanes are used, a position vector to each mainsail beam tip will be reported, as well as the attitude of each mainsail beam tip relative to the sailcraft body-fixed Sail system. The Euler sequence for rotation of a Beam Tip system relative to the Sail system is:

```
"Index" about z:

0° for the fore beam tip
90° for the starboard beam tip
180° for the aft beam tip
270° for the port beam tip
then:

"Bend" about y -> "Sway" about z -> "Twist" about x
```

Positive sense is determined by the right-hand rule. The transformation matrix for a vector [V] between Sail and Beam Tip systems is:

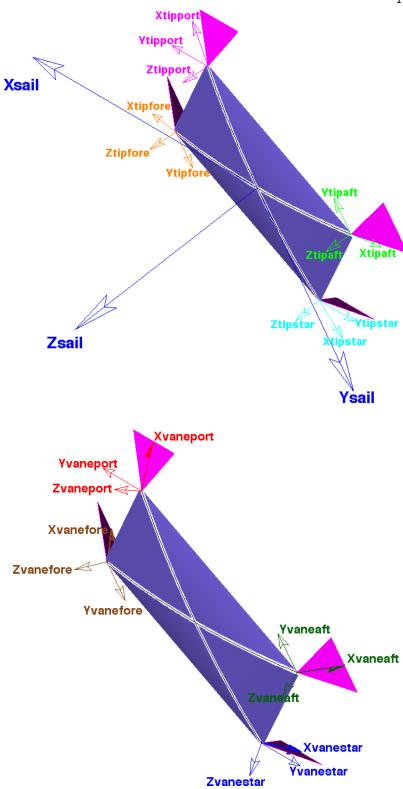


Figure 7 Beam Tip and Vane Coordinate Systems

Specification of Vane Rotations Relative to the Beam Tip

If vanes are used, two types of 2-axis vane yoke designs are considered. They are identified by the Vane Euler rotation sequence relative to Beam Tip as either:

Positive sense is determined by the right-hand rule.

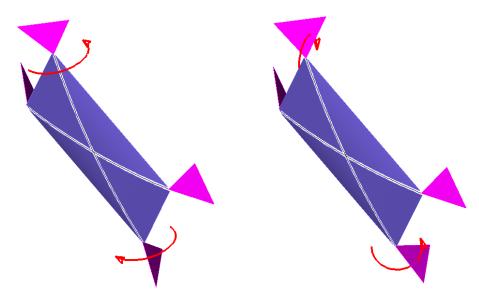


Figure 8 "Twirl->Cant" Type 2-Axis Yoke

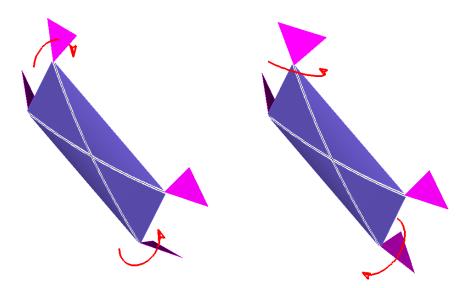


Figure 9 "Cant->Twirl" Type 2-Axis Yoke

It is assumed that yoke design will be such that there is no significant offset between the Twirl and Cant axes. The Beam Tip system origin is coincident with the Vane system origin. The mass of the yoke and actuators is included in the mainsail.

The transformation matrix for a vector [V] between Beam Tip and Vane systems is:

```
[V]_{Vane} = [R]_{V-T} [V]_{Tip}
```

for Twirl about x -> Cant about y:

for Cant about y -> Twirl about x:

Vane performance will be reported separately from mainsail performance. Vane tables will actually specify Vane Sun Incidence and Flatspin angles relative to the sun, which the user must calculate from vehicle attitude, beam tip attitude, and user-desired Vane Twirl & Cant. VaneSunIncidence is the angle between the vane normal and the sail-sun line. The equation for VaneSunIncidence (VSI) is developed using the dot product of a unit vector Zsun along the sail-sun line and a unit vector Zvane:

```
Zsun • Zvane = cos(VSI) |Zsun| |Zvane| VSI = acos(Zsun_{vanez})
```

The dot product will always return a [positive] included angle. The returned angle will further be limited to the luff limit of the vane (Zsun_{Vanez} will be positive). The lack of negative values of VSI is OK as long as VSI and Vane Flatspin (VFS) are not used to derive attitude of the vane relative to the Beam Tip, only for coefficient lookup.

To express Zsun in the Vane coordinate system, transform from Sun to Sail to Tip to Vane. The appropriate matrix must be chosen for Twirl -> Cant vs. Cant -> Twirl:

```
[Zsun]_{vane} = [R]_{V-T} [R]_{T-S} [R]_{S-Su} [0,0,1]

VSI = acos(\{[R]_{V-T} [R]_{T-S} [R]_{S-Su} [0,0,1]\}_{z})
```

In order to find VaneFlatspin (VFS), take the dot product between the projection of Zsun onto the Vane x-y plane and a unit vector along the negative Vane X-axis (the negative axis because VSI as calculated above will always be positive):

```
\begin{split} & \operatorname{Zsun}_{vane \ x-y} \bullet \operatorname{-Xvane} = \cos(VFS) \ | \operatorname{Zsun}_{vane \ x-y}| \ | \operatorname{-Xvane}| \\ & \operatorname{VFS} = \operatorname{SIGN}(\operatorname{zsun}_{vaney}) \ \operatorname{acos} \{ \operatorname{-Zsun}_{vanex} \ \div \ \sqrt{(\operatorname{Zsun}_{vanex}^2 + \operatorname{Zsun}_{vaney}^2)} \} \\ & [\operatorname{zsun}]_{vane} \ = \ [R] V-T \ [R] T-S \ [R] S-Su \ [0,0,1] \\ & \operatorname{SIGN}(\operatorname{zsun}_{vaney}) = \operatorname{zsun}_{vaney} \div |\operatorname{zsun}_{vaney}| \end{split}
```

The assignment of sense is necessary because the dot product will only return an angle between 0° and $+180^{\circ}$. Note that this assignment again relies on VSI being positive, as calculated above.

As mentioned earlier, if $VSI \approx 0^{\circ}$ (the vane is ~normal to the vane-sun line), the user should skip these equations and return "N/A" (or equivalent) for VFS. Otherwise, the dot product may have an x/0 error.

Specification of Gimbaled Mass Relative to the "Sail" System

If a gimbaled mass such as an antenna is used, the mass of the gimbal and actuators will be included in the mainsail. The gimbaled mass itself will be reported separately from the mainsail mass, and will be assumed constant (no significant expendables). Several such gimbaled masses may be included, as with an instrument boom pointing toward the sun, together with an antenna pointing toward the ground.

There are two categories of gimbaled masses, those which can be pointed for control purposes, and those which cannot. The payload is generally divided into sail-fixed mass and control-gimbaled mass, both of which can be modified by the user.

The position vector from the Sail system origin to the gimbal will be reported in the Sail system. The position of the gimbaled mass center, as well as its moments of inertia, will be reported in a gimbaled mass-fixed frame. This frame will have its origin at the gimbal, its Z-axis pointing generally along positive or negative Zsail (to be indicated by the sail designer). The position of such a frame relative to Sail will be described using the convention of "Azimuth" and "Elevation," with "Phase" describing any final rotation about the Zaxis of the gimbaled mass. The positive sense and reference for these angles will be described by the sail designer.

FACTORS AFFECTING PERFORMANCE

Shape Effects Due to Solar Distance and Centralized Mass

The closer the sail is to the sun, the more the propulsive load transfer to the centralized payload and bus, therefore the more the beams bend. This changes sail shape somewhat, so solar distance must be specified in the tables of coefficients. Also, closer distance to the sun will amplify the effects of solar disc diameter and limb darkening.

Sail designs will differ for different minimum design solar approaches. The minimum solar loading (maximum solar distance) a sail can be structurally designed to handle is of course at 1.02 AU (Earth Aphelion), where the sail is deployed and has to bear load. Sails designed to fly to Mercury at ~0.3 AU, for example, will need stronger (and therefore heavier) structures in order to preserve structural safety factor, as well as more emissive backside sail coatings. These will affect propulsion and deflected shape across all solar distances. Minimum solar approach is a design variable, like sail area or assumed packaging volume available, as opposed to one which may vary over a trajectory, or just using larger vanes or a heavier payload with the same sail, thus will be presented as an entirely different set of tables. Structural safety factor at the design minimum solar approach will also be reported for fair comparisons.

Payload mass will also affect sail shape and therefore forces and moments. This is because the heavier the centralized mass, the less distributed the load will be and the more "cantilevered" the beams will be, thus deflecting more and altering sail shape. A user may desire to increase or decrease payload mass, therefore the sail designer must report the payload mass assumed in structural shape calculations, along with the accuracy that a user can expect by varying the payload mass a certain percentage. The mainsail mass properties will not include any payload mass. The user will select his own payload mass based on mission needs, and calculate the combined mass properties of the sailcraft himself. A user may request a sail designer recalculate shape and report new coefficients using his specific payload mass, in order to improve accuracy.

The position vector to the payload center of mass will be given, and will be assumed constant with user variations in payload mass. If part or all of the payload is control-gimbaled, the gimbaled portion will be described separately, in the gimbaled mass frame.

In the current format, it is assumed that from a mass standpoint no significant expendables or ballast are used, and all parasitic masses that could be jettisoned have already been released. All masses are constant.

Effect of Vanes or Gimbaled Masses on the Mainsail Shape

Vane solar, inertial, gravitational, and dynamic actuation loads affect mainsail beam tip deflections, therefore mainsail shape and propulsion. Gimbaled masses also apply forces and moments to the mainsail, and their actuators apply torques.

All sails with the center of mass forward of the center of pressure experience some passive stability about the two in-plane axes due to the solar "drag" force of an imperfectly reflecting sail, and sail billow imparts more stability due to the effective shuttlecock angle. Vanes may also be canted anti-sunward for added strong passive stabilization, and vane neutral angles can be reset to stabilize about an axis not in the sail plane. Vane or control mass angles at which there is no net moment about a certain axis on the vehicle for a given vehicle attitude are called "trim" settings for that axis. Note that net vane or control mass forces on the mainsail will still exist with the vehicle trimmed. Internal moments may also exist, affecting shape. Various combinations of vane angles can achieve trim, with different internal moments.

The sail designer will report separate solar propulsion performance tables for the mainsail alone, and for various detached single vanes (if used). Sail designers using gimbaled control mass will provide various mast length designs. Mainsail, vane, and gimbal mass properties are reported separately. A certain vane or gimbaled control mass will be assumed when the sail designer calculates mainsail shape (therefore propulsion), but the solar, inertial, and gravitational forces on the vanes or control masses themselves will not be included in reported mainsail propulsion. The vane or control-gimbaled payload mass assumed in mainsail shape calculation will be reported, along with the error associated with using a vane a certain percentage larger or a control-gimbaled payload mass a certain percentage heavier or a longer control mast. This will allow the user to mix and match mainsails with control devices of various sizes. Combined forces, moments, and mass properties for the assembled sailcraft will be calculated and transformed by the Once a user has determined an appropriate vane or control mass size for his particular GN&C design and mission, he may ask the sail designer to recalculate mainsail shape and report new coefficients using these exact control sizes, in order to improve accuracy. This iteration may reoccur several times.

In calculating mainsail shape, the sail designer will not include any torques due to actuation, nor any actuator rates. The max allowable reaction torque will be supplied. Also, when a sail designer calculates sail shape at an off-trim attitude, he will assume zero inertial rotation rates. The sail will be undergoing rotational acceleration due to the net moment, but at zero angular velocity, steady state. Note that the sail is always undergoing translational accelerations, due to the imbalance and angles between the solar gravity and solar pressure. The coefficients will thus be static; transient dynamic

response of the [flexible] sail will not be represented. If a user wishes to investigate a spinning sail, he may ask the sail designer to recalculate sail shape assuming a constant nonzero rotation rate about some axis, and report a new set of coefficients.

It would be impractical and unnecessary for a sail designer to calculate mainsail shape at all possible combinations of vehicle attitude and vane or control mass angles. As a minimum, sail designers will determine mainsail shape at each attitude with the sail trimmed, and report the vane or gimbaled mass angles used to achieve trim at each sail attitude. The assumed yoke design (Twirl->Cant vs Cant->Twirl) for a vane must also be given. Users may later request accurate coefficients with the vanes at different angles that still achieve trim. An example would be if a GN&C designer desires greater passive stability, he would ask for mainsail coefficients to include the shape effect of all 4 vanes canted further anti-sunward. Users may also request accurate coefficients in an off-trim condition, such as after an attitude perturbation that drove the vehicle off-trim, or with vanes or control mass rotated to effect a maneuver.

Factors Affecting Vane Shape

The vane will bend and billow as a structure as well, so major parameters affecting its shape must be reported. Solar distance will affect loading and bend, and is therefore reported. Also, just as the mainsail shape is affected by heavier centralized masses, so the vane is affected by what mainsail it is attached to. Something more than mass is required, however, as the sail is propulsive. The lightness factor of the sailcraft assumed in the vane structural analysis will be reported in the vane performance table.

Momentum-Producing Biases

Sail shape will vary from ideal due to manufacturing errors, deployment effects, and other factors. These effects, along with asymmetric shape effects already represented in the coefficients, will give rise to bias moments on the sailcraft. These will generally be a function of solar distance and attitude, as they ultimately result from solar pressure. They need to be trimmed out, or the resulting momentum dumped. It is not possible to passively trim Top disturbances, as there is no inertial reference about the sun-sail line. Active stabilization must be used. It is therefore important for the sail designer to determine how much bias moment might be developed, especially about the sail-sun line.

Two approaches are taken. The sail designer will supply estimates of estimated average Top bias moment vs solar distance. He will also postulate defects in the sail due to manufacturing, deployment, and the like, and generate entirely new tables of coefficients for the corrupted sailcraft.

Reflectivity Degradation

There is some belief that the reflectivity of certain aluminized sail materials may degrade with extended exposure. Two approaches are again taken. The sail designer will provide estimates or data (if available) on the total reflectivity of his sail material vs. accumulated dose. The user will apply this degradation to zero-dose coefficients in a bulk manner. The sail designer will also supply a set of tables, one for each total dose, with the correct reflectivity used in coefficient calculation.

Rigid Body Damping

It is believed that structural damping will result in some rigid body damping of the sailcraft. Sail designers will supply estimates of rigid body damping coefficients.

Attitude & Angle Limitations and Fault Tolerance

Sails will generally have Sun Incidence limits for safe operation. One practical limit is that at which the max camber line of an aft sail quadrant becomes shadowed, the "luff limit," which will be reported. Tighter limits may be specified by the sail designer as well for other reasons.

Vanes and gimbals will generally have practical limits, as well as mechanical stops put in place to prevent oversteering. These will be reported, along with any "watchdogs" implemented for safe operation, such as hardwire logic observing vane angles and vehicle attitudes and rates. Sail designers should also postulate and report credible faults affecting attitude control.

NON-DIMENSIONALIZED COEFFICIENTS OF FORCE AND MOMENT

A force coefficient is non-dimensionalized as follows:

force coefficient "Cf" = F/[PA] = force / [solar pressure at distance used in calculation X nominal projected area] solar pressure = insolation (W/m²) / speed of light (m/s) "nominal" projected sail area is held as a constant

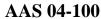
The idea is that these coefficients could be applied within a range about the solar distance used to come up with the beam bend, without affecting results much. Also, different people use different values for solar insolation, so the format tries to take out sensitivity to this.

Mainsail moments are given about the "Sail" system origin; vane moments are about the "Vane" system origin; a gimbaled mass moment about its system. A moment coefficient is non-dimensionalized as follows:

moment coefficient "Cm" = M/[PAL] = moment / [solar pressure X nominal area X reference length] The reference length will be the square root of the nominal projected area

The tables will also include a position vector to the mainsail, vane, and gimbaled mass centers of mass, as well as the moments of inertia about the "Sail" origin for the mainsail; about the "Vane" origin for the vane, about a gimbaled mass's own system. Payload centers of mass are described separately.

Given accurate points, the user will develop means to interpolate between. An example of what this might look like can be demonstrated using a polar plot of 1 AU flatsail thrust vs. thrust "cone" (Ref. 1) angle. Polar plots are commonly used to specify performance of racing sailboats, whose speed vary with course.



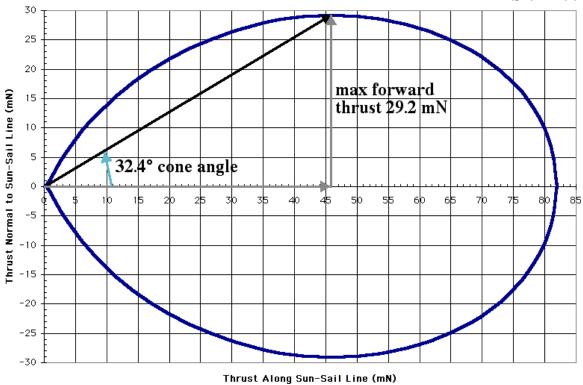


Figure 10 Polar Plot of 10,000 m² Flatsail Thrust at 1 AU vs. Cone Angle (Sun is on the Left)

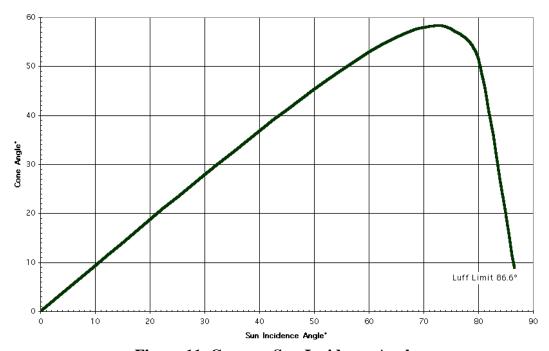


Figure 11 Cone vs. Sun Incidence Angle

Estimated Error

Estimates of the error between reported coefficient values and what will actually be experienced in space will be supplied by the sail designer. This will allow a user to design and utilize smartly. For example, a GN&C designer will be able to decide the most reliable, perhaps not the highest performing, scheme for thrust vectoring.

The designer will also identify the analysis models used to generate the tables, under "modeling accuracy." For example, early tables may assume the sun is a point source, or they may assume a sail billow which is independent of attitude or solar distance. Any data relied on from ground test or in-space calibration should be identified in this section as well.

LIFE REPORTING

A user must know how long he can operate the sail. Sail designers will report factors that limit life. One of these is the use of expendables, such as to dump momentum, or the use of primary batteries. If so used, the approximate average expenditure rate must be reported, as a function of whatever it may be strongly dependent on, such as solar distance. Note that expendable mass is assumed insignificant and not accounted for from a mass standpoint, but its effect on life is modeled.

Beam and sail materials can also limit life. Material life is generally a function of accumulated particulate and UV doses, which the user will calculate for the desired trajectory. Maximum accumulated doses before failure vs AU will be given in the tables. Any information known about combined effects should also be given.

Sailcraft avionics contractors will supply estimates of component life.

LAUNCH PROPULSION REQUIRED (PACKAGING PERFORMANCE)

The cost of an Ariane µASAP launch is currently \$3M - \$4M, perhaps another \$1M for an Earth escape kick motor. Another commonly proposed launcher for solar sails is the Delta II, at least an order of magnitude higher in cost at \$40M - \$50M. Discussion of the propulsive performance of a sail cannot go without an evaluation of the propulsion needed to launch it, which comes down to packaging volume, as gossamer structures will generally be volume-limited, rather than mass-limited for launch.

This will only come into play in the trajectory as initial conditions from the launcher, but packaging performance should always be reported with any performance

numbers for fair comparisons between sails, as well as to clearly identify a specific sail design. Packaging accommodations do actually affect the sailcraft design itself. Other cost drivers will be reported separately to interested program managers.

The driver in sailcraft packaging generally turns out to be the length and cross-section area of the packaged beams. A large deployed cross-section is necessary for structural stiffness, but can consume much of the packaging volume if not collapsed somehow.

The following packaging performance parameters will be reported:

Assumed Launch Slot and Lift Capability Earth Escape Kick Motor (if any), and Kick Capability Payload & Bus Packaging and Mass Limitations

Sailcraft Packageability (Deployed Sail Area ÷ Stowed Volume of Sail + Beams) Packaged Volumetric Density of Sail + Beams

Beam Length Contraction Ratio (= deployed length ÷ packaged length)

Packaged Beam Volume % of Deployed (= packaged beam volume ÷ deployed)

Beam Effective Cross-Section Area Ratio (= [1/Volume Percent] ÷ Length Ratio)

Launch vehicle utilization, sailcraft packageability, and sailcraft mass performance are related as:

Volumetric Density (g/cc) X 1,000,000 = Packageability (m^2/m^3) X Areal Density (g/m^2)

CONCLUSION

We hope that this specification allows accurate and sensible representation of the predicted performance of gossamer solar sail structures. This is intended to be an open specification, for which this is the first version. Please feel free to contact the authors with suggested changes or questions for clarification.

REFERENCES

1) McInnes, C.R., Solar Sailing Technology, Dynamics and Mission Applications, Springer-Praxis, London, UK, 1st Ed.